DSC-Vol. 31, Modeling and Control of Compliant
and Rigid Motion Systems
ASME 1991

TRACKING CONTROL OF A NONMINIMUM-PHASE FLEXIBLE MANIPULATOR

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Abstract

Perfect tracking or asymptotic tracking of the tip position of a flexible manipulator, which is a typical nonminimum-phase system, has been very difficult because of the positive zeros of the nonminimum-phase system and the lack of desired trajectories of flexible coordinates. This paper presents a tracking control scheme combined with the inverse dynamic feedforward control. The inverse dynamic method calculates the feedforward torque that cancels poles and zeros of the nonminimum-phase system. It also generates the desired flexible coordinate values, which match equivalent to the tip position trajectory dynamically. The feedback loop achieves tracking capability with the calculated desired flexible coordinate trajectories. The control scheme has been applied to the tip position control of a single-link flexible manipulator for zero and non-zero initial condition cases. The non-zero initial conditions of the system states are divided into three components of rigid body, causal and anticausal parts. The rigid body component is used for the desired tip trajectory planning and the other components of these are used separately for the calculation of the feedforward torque of causal and anticausal parts. Through simulation and experiment, we explore the effectiveness and limitations of this method for moving non-zero initial condition cases.

1. Introduction

More and more today, robot manipulators are being used in the manufacturing processes, such as welding, material handling, and assembly. An increasing number of applications demand the capability of larger work-space, more accurate positioning, faster motion, less power consumption, and greater payload. The lightweight, large work-space manipulator has intrigued the engineers who have to deal with large objects, such as the space station assembly, large structure welding, airplane assembly, and nuclear waste handling. However, the large lightweight structure results in inherent flexibility, which induces complex dynamics in a manipulator.

The dynamic model is infinite dimensional due to infinite flexible modes. It is nonlinear, and always has uncertainty due to the unmodeled dynamics. The tip position control with the joint torque is usually nonminimum-phase due to flexibility, and noncolocated because of different sensor/actuator location. In spite of the above characteristics of a flexible manipulator, the end-effector position should accurately track the desired path for some practical applications. Therefore, our control objective becomes the tracking control of the nonminimum-phase, noncolocated, and nonlinear system with uncertainties. This may be the most general and complicated problem in the control of robotic manipulators.

To design a proper control scheme for a flexible manipulator, we have to decide what to overcome, what to avoid, or what to compromise. Most recently proposed ideas for the control of a flexible manipulator can be classified into following categories. Each method seems to have its advantages and penalties.

a) Use many sensors and actuators

If the number of actuators and sensors are equal to the number of controlled modes, the flexible structure can be controlled easily without a spillover problem [14]. Crawley [8] showed the possibility by using a distributed array of piezoelectric devices for actuation on a flexible structure. This may be the direct approach to the multiple mode system, but the penalty is the hard ware cost.

b) Change the structure of the manipulator

Alberts, et al [1] used a thin film of viscoelastic material between the structure surface and an elastic constraining layer to obtain passive damping. Park and Asada [15] tried to change the nonminimum-phase flexible manipulator system to a minimum-phase system by relocating the torque actuation point. This approach may be limited because the modification of the structure is not always possible.

c) Use advanced feedback control schemes

Hastings and Book [9] used optimal control with the strain feedback. Cannon and Schmitz [6] demonstrated the end-position control with the tip position and the joint rate feedback. Kotnik, et al [11] presented the results of the end-point acceleration feedback. Wang and Vidyasagar [19] applied the stable factorization approach to obtain the optimal step response. Singular perturbation methods were tried by Siciliano and Book [16], Khorrami and Ozguner [10], and others. Various adaptive techniques have been proposed by Yuh [21], Yuan and Book [20], Cetinkunt and Wu [7], and other researchers. Bartolini and Ferrar [3] tried to extend the adaptive pole assignment control to nonminimum-phase system. Even though many control schemes have been proposed to control the flexible vibration actively, most control schemes will allow bounded tracking errors because the feedback control cannot cancel the nonminimum-phase zeros that have positive real values.

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have negative eigenvalues and T_{ac} is made of the eigenvectors of positive eigenvalues.

$$[T]^{-1}A_i[T] = \begin{bmatrix} A_{ic} & 0 \\ 0 & A_{iac} \end{bmatrix}$$

$$\begin{Bmatrix} \dot{P}_c \\ \dot{P}_{ac} \end{Bmatrix} = \begin{bmatrix} A_{ic} & 0 \\ 0 & A_{iac} \end{bmatrix} \begin{Bmatrix} P_c \\ P_{ac} \end{Bmatrix} + \begin{bmatrix} B_{ic} \\ B_{iac} \end{bmatrix} \dot{q}_{ir} \tag{9}$$

$$\begin{Bmatrix} \tau_c \\ \tau_{ac} \end{Bmatrix} = \begin{bmatrix} C_{ic} \\ C_{iac} \end{bmatrix} \begin{Bmatrix} P_c \\ P_{ac} \end{Bmatrix} + \begin{bmatrix} 1/2\ F_i \\ 1/2\ F_i \end{bmatrix} \dot{q}_{ir}$$

$$\tau = \tau_c + \tau_{ac}$$

Such a coordinate change decouples the inverse system into two subsystems of Eqn. (9). The new variable P_c represents the coordinates of the causal system, and the P_{ac} represents the anticausal system.

For a given end-point trajectory of Figure 2, the causal part torque is obtained by integrating the causal part inverse dynamic equations forward from the initial time of the trajectory. Then, the anticausal system equations must be integrated backward from the final time of the trajectory. The total torque, which is the output of these equations, is obtained by adding both torques as shown in Figure 3. Those state variables of the desired motion that were unspecified directly by the desired tip position are prescribed by the inverse dynamics as a function of time. These functions are called desired flexible coordinate trajectories. They are physically either mode amplitudes and their derivatives or equivalently strain at points on the beam and their derivatives.

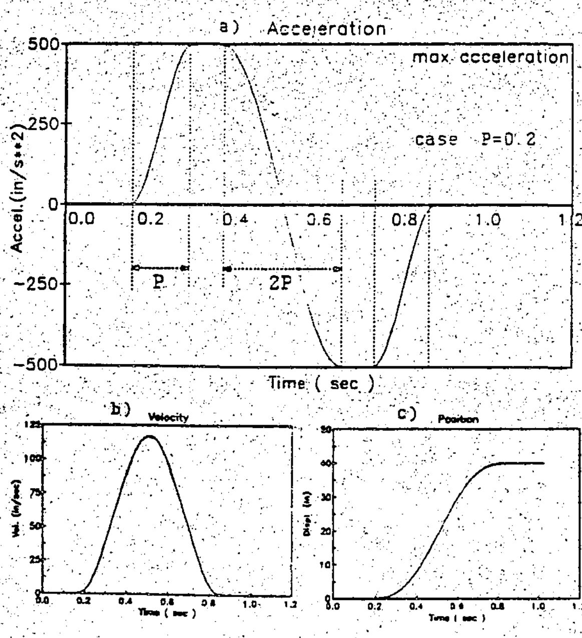

Figure 2. Desired end point trajectory a)Acceleration, b)Velocity, c)Position

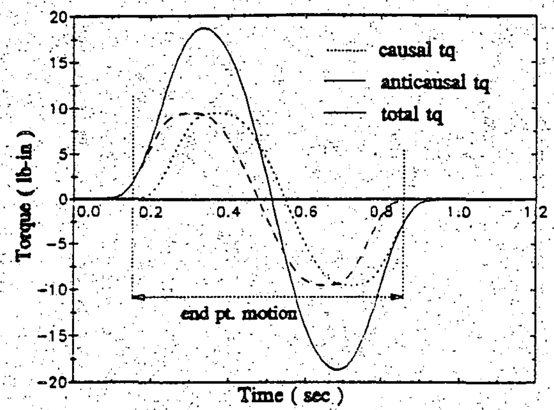

Figure 3. Calculation of torque with the inverse dynamic method

4. Analysis of the System State Space

It is very useful to analyze the relations between the dynamic system states, and causal and anticausal subsystems' states to construct additional desired trajectories of flexible coordinates. We can obtain the reference trajectory of the flexible mode coordinates of the direct dynamic system from a given rigid mode trajectory and the calculated state trajectories of the causal and anticausal systems.

As we can expect in Eqn. (2), (8), and (9), the space of the full state vector X of the direct dynamic system can be divided into three subspaces: the rigid body coordinate subspace q_{ir}, the causal part flexible coordinate subspace P_c, and the anticausal part flexible coordinate subspace P_{ac}. These subspaces are linearly independent and orthogonal to one another. The relations of these spaces are illustrated in Figure 4, and described by the following Eqn. (10) when only two flexible modes are considered.

$$\textit{where}\quad X = \{q_r,\ q_{f1},\ q_{f2},\ \dot{q}_r,\ \dot{q}_{f1},\ \dot{q}_{f2}\}^T$$

$$q_{ir} = \{q_r,\ \dot{q}_r\}^T,\quad X_i = \{q_{f1},\ q_{f2},\ \dot{q}_{f1},\ \dot{q}_{f2}\}^T$$

$$X = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} q_{ir} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} X_i \tag{10}$$

$$X = H_r q_{ir} + H_f (TP_i)$$

$$X = H_r q_{ir} + H_f T_c P_c + H_f T_{ac} P_{ac}$$

From the given end point trajectory, the rigid body coordinate trajectory q_{ir} is obtained. The flexible coordinate trajectories of P_c and P_{ac} are then calculated from the integration of Eqn. (9). Thus, the trajectories of the complete state vector X can be obtained by using Eqn. (10). These trajectory values can be used as reference

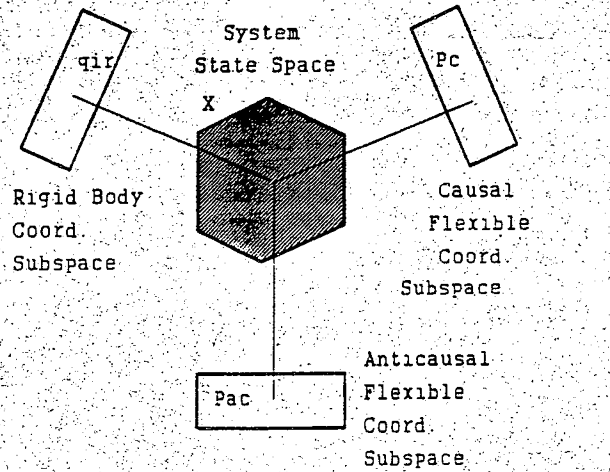

Figure 4. Dimensional analysis of state variables of flexible manipulator dynamic equations

commands for feedback tracking control.

The generation of complete state trajectories is an advantage of this inverse dynamic method. We get the strain trajectories as well as the joint trajectory. We no longer have to give the flexible manipulator reference commands to follow the trajectory like a rigid manipulator.

5. Tracking Controller Design

Let's consider a linear nonminimum-phase model (the output is the tip position), and a linear minimum-phase model (the output is the joint angle) of a flexible manipulator in the form of following transfer functions.

$$\textit{nonminimum-phase} \qquad \textit{minimum-phase}$$

$$X_e(s) = \frac{Z_e(s)}{P(s)}\,\tau(s)\,, \qquad \theta(s) = \frac{Z_\theta(s)}{P(s)}\,\tau(s) \tag{11}$$

where X_e : *Tip position*
θ : *Joint angle*
τ : *Joint torque input*

$$P(s) = s^2 \prod_{i=1}^{n} (s \pm j p_i)$$

$$Z_e(s) = \prod_{i=1}^{n} (s \pm z_{ei})$$

$$Z_\theta(s) = \prod_{i=1}^{n} (s \pm j z_{\theta i})$$

The nonminimum-phase system has imaginary poles p_i and real positive and negative zeros z_i, while the minimum-phase system has the same imaginary poles p_i and imaginary zeros $z_{\theta i}$.

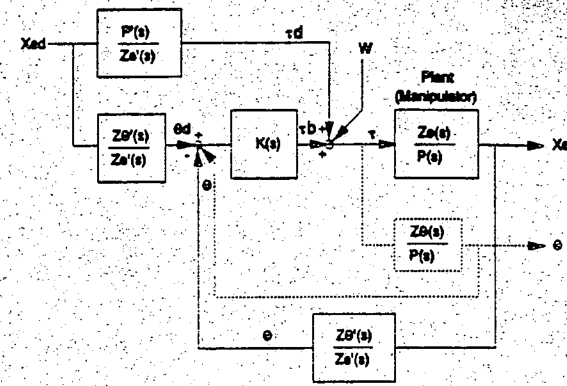

Figure 5. Tracking control of a nonminimum-phase flexible manipulator

The control objective is to make the output $X_e(t)$ (the tip position) follow the desired time-varying trajectory $X_{ed}(t)$ applying the joint torque $\tau(t)$. In Figure 5, the input torque τ will consist of the inverse dynamic feedforward torque τ_d, the feedback torque τ_b driven by tracking errors, and the disturbance or friction torque w. P(s), $Z_e(s)$, and $Z_\theta(s)$ represent the poles of the system, zeros of the nonminimum-phase system, and zeros of the minimum-phase system respectively. P'(s), $Z_e'(s)$, and $Z_\theta'(s)$ are the estimated models of the real system.

The feedforward torque τ_d is obtained using the inverse dynamic method that is equivalent to the two-sided Laplace transform with the region of convergence along the imaginary axis to get the stable solution in the time-domain. The feedback torque τ_b is generated by the error signals between the desired joint and strain trajectories, and the measured joint angle and strains. In experiment, the joint angle is measured directly, not estimated from the end-point position X_e. Therefore, in Figure 5, the actual feedback loop is the dotted line (...) rather than the solid line (—). Using the colocated feedback control with the joint trajectory, we avoid the noncolocated control problem of the direct the tip position feedback. This advantage is obtained from generation of the desired flexible coordinate trajectories, which gives the joint trajectories and the strain trajectories that match the tip trajectory dynamically.

The equations that implement these descriptions appear below, and they are also displayed in the block diagram in Figure 5. The flexible manipulator motion is represented by the Eqn. (12).

$$X_e(s) = \frac{Z_e(s)}{P(s)}\,\tau(s) \tag{12}$$

$$\textit{where} \qquad \tau = \tau_d + \tau_b + w$$

$$\begin{aligned}
\tau_d &= \frac{P'(s)}{Z_e'(s)}\,X_{ed}(s) \\
\tau_b &= K(s)\,(\theta_d(s) - \theta(s)) \\
\textit{where} \quad \theta_d(s) &= \frac{Z_\theta'(s)}{Z_e'(s)}\,X_{ed}(s)\,, \\
\theta(s) &= \frac{Z_\theta(s)}{P(s)}\,\tau(s) \quad \textit{or} \quad \frac{Z_\theta'(s)}{Z_e'(s)}\,X_e(s)
\end{aligned} \tag{13}$$

From Eqn. (12), (13), the output $X_e(s)$ will be in the form of

$$X_e = \frac{Z_e}{P + KZ_\theta} W + \frac{Z_e(P' + KZ'_\theta)}{Z'_e(P + KZ_\theta)} X_{ed} \tag{14}$$

If the feedback gain of K(s) becomes very large, the disturbance effect will be negligible, so that the tracking performance will be depend on the accuracy of the zeros of the model as follows.

$$X_e = \frac{Z_e Z'_\theta}{Z'_e Z_\theta} X_{ed} \tag{15}$$

From Eqn. (14), the tip position tracking error dynamics will be as follows:

$$(1 + \frac{KZ_\theta}{P})E = \frac{Z_e}{P} W + \{(\frac{P'Z_e}{PZ'_e} - 1) + \frac{K}{P}(Z_e \frac{Z'_\theta}{Z'_e} - Z_\theta)\} X_{ed} \tag{16}$$

$$where \quad E = X_{ed} - X_e$$

If the disturbance effect is negligible, and the dynamic model is good enough to cancel the system dynamics, the error dynamics will converge to zero as shown in Eqn. (17).

$$(1 + \frac{KZ_\theta}{P})E = 0 \qquad If \;\; W = 0$$

$$\begin{aligned} P' &= P \\ Z'_e &= Z_e \\ Z'_\theta &= Z_\theta \end{aligned} \tag{17}$$

Then, the tracking dynamics are determined entirely by the joint feedback loop characteristics, which is a stable colocated control loop. Therefore, the control scheme will guarantee perfect tracking if the initial tracking errors are zero, and will guarantee asymptotic tracking convergence for non-zero initial tracking errors, as long as the model is accurate. Even if the model is not exact, the tracking error will be bounded because the positive poles of the inverse system give a stable torque profile using the two-sided Laplace transform. The effect of model inaccuracies is not the principal subject of this paper. However, 50% end-mass variations (equal to 1.6% of total link mass) showed negligible bounded tracking errors, but induced residual vibrations at the stop motion. The effects of the unmodeled dynamics, joint friction, and payload variation are subjects for future study. [22]

6. Perfect Tracking with Zero Initial Conditions

The above combined control scheme of the inverse dynamic feedforward control and the feedback control was implemented for a rest-to-rest motion of the experimental manipulator. For a real time control, a Micro Vax II was used with 12 bit A/D and D/A boards. The off line calculation of a trajectory and a torque profile was also performed in the Micro Vax.

By applying the precalculated torque, compensating the joint friction, and using the feedback of the tracking error at the joint, the excellent result of Figure 5.5 was obtained. The flexible manipulator could stop without any overshoot or any residual vibration after it moved 40 inches (48.76 degrees) within less than 0.8 second. In the strain signal, there exists a rough jerk that could be eliminated by using a smoother acceleration profile. Unfortunately, since an end-point position sensor was not available for the system, the end point position couldn't be measured directly. However, the end point tracking performance can be estimated from the joint tracking result and the strain tracking result. If the joint doesn't have any overshoot or vibration and the strain doesn't show any residual vibration, the end point can be presumed to stop without any overshoot or vibration.

The experimental results show that a simple joint feedback PD controller provides excellent tracking if it is combined with the inverse dynamic feedforward control, and if the joint trajectory are generated from the desired end-point trajectory considered flexible dynamics. Full state feedback has been shown to converge even faster, but the increased difficulty in implementation seems unwarranted.

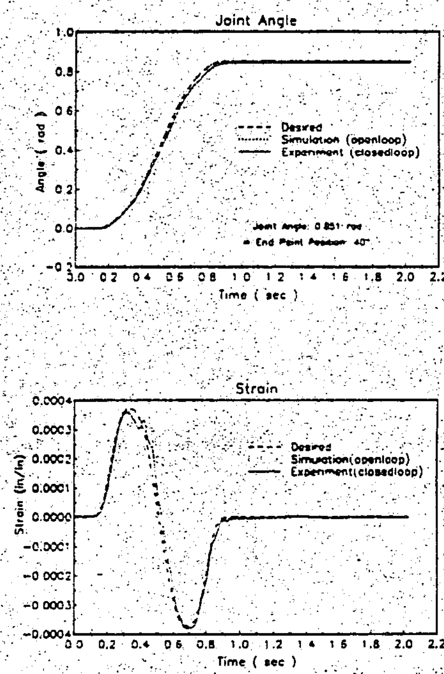

Figure 6. Experimental results of the combined control of the inverse dynamic feedforward control and the joint tracking feedforward control: a) Joint angle, b) Strain at the base

7. Tracking Control for Non-zero Initial Condition Cases

The previous section showed good tracking results for rest-to-rest motion, which gives zero initial and final flexible coordinate values. How, then, can this method can be applied to non-zero initial condition (moving) cases? In the control scenario of Book's bracing manipulator [13], the large flexible manipulator will be commanded to move in free space using a simple feedback controller with rough information of the target surface position until it senses the target bracing surface. If it measures more accurately the distance from the tip position to the bracing surface during motion, the controller will modify or create the desired tip trajectory accommodating the moving non-zero initial conditions, and calculate the required torque and the desired trajectories of all states with respect to the new tip trajectory. Then, it will follow the trajectory and will stop just in front of the surface or approach the surface with a suitable slow speed, and will contact to the surface without a large impact. Therefore, we need to be able to do trajectory planning and tracking control for general moving initial conditions.

7.1 Analysis of Initial Conditions of the States and Trajectory Planning

As explained in Figure 4 of section 4, the system states X of a flexible manipulator's dynamics can be divided into a rigid body component q_{ir}, a causal component P_c, and an anticausal component P_{ac}. Naturally, the initial system states X_0 can be divided into q_{ir0}, P_{c0}, and P_{ac0}.

$$X_0 = H_r q_{ir0} + H_f T_c P_{c0} + H_f T_{ac} P_{ac0}$$

First of all, let's assume the initial condition (I.C.) X_0 and the desired final state X_f is known or measured exactly. The final condition X_f also will be divided into three components q_{irf}, P_{cf}, and P_{acf}. From the initial and final values of the rigid body coordinate, we are generating the desired end point trajectories including the desired acceleration, velocity, and position profiles considering the minimum traveling time and the high frequency content. The desired trajectories of Figure 7 are generated with the following example data: the end point initial acceleration $X_{ea0} = -10$ in/s^2, the initial velocity $X_{ev0} = 60$ in/s, the initial position $X_{e0} = 16.4$ in, and the desired final position $X_{ef} = 57.4$ in, $X_{evf} = 0$, $X_{eaf} = 0$. The final condition doesn't have to be stationary: it can be an arbitrary moving condition. Now we have planned the desired tip trajectories of a flexible manipulator. If the tracking controller is working properly, the end point will follow the trajectory and will stop without any overshoot or vibrations.

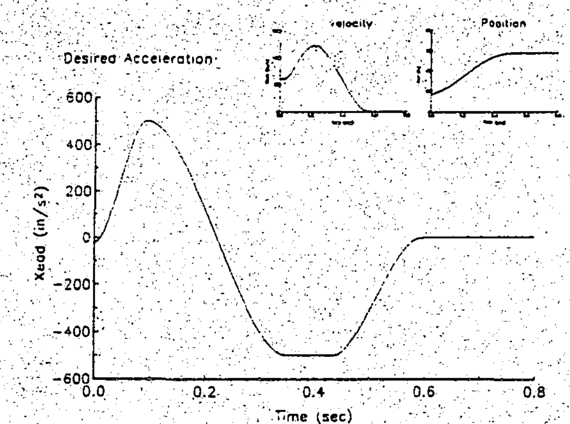

Figure 7. The desired end-point trajectories for non-zero I.C.

7.2. Inverse Dynamic Torque Calculation

Since the initial and final conditions, and the rigid body coordinate trajectory (equivalent to the end-point trajectory) are given, the torque profile can be calculated using the same inverse dynamic method for the zero I.C. case. The causal torque is calculated by integrating the causal part equation with the initial condition P_{c0}, and the anticausal torque is calculated by integrating the anticausal part equation backwards with the final condition P_{acf} as shown in Figure 8. That means the anticausal part of the I.C. P_{ac0} cannot be considered in the inverse dynamic torque calculation. This is a limitation of the inverse dynamic method for non-zero initial condition case with the current fixed acceleration profile of the end-point. If the acceleration has more degree of freedom with extra parameters, it can satisfy the anticausal part initial condition by manipulating the auxiliary parameters. However, this approach of changing the acceleration profile structure is not so practical if implemented. In this research, we maintain the acceleration profile of Figure 2 adjusting the acceleration and deceleration time ratio without trying to match up to the anticausal part I.C. Even though the initial conditions of anticausal part flexible coordinates cannot be considered in feedforward inverse dynamic control, the tracking error can be made to converge to zero by the asymptotic tracking capability of the feedback loop. The following sections examine several mismatched cases of initial tracking errors.

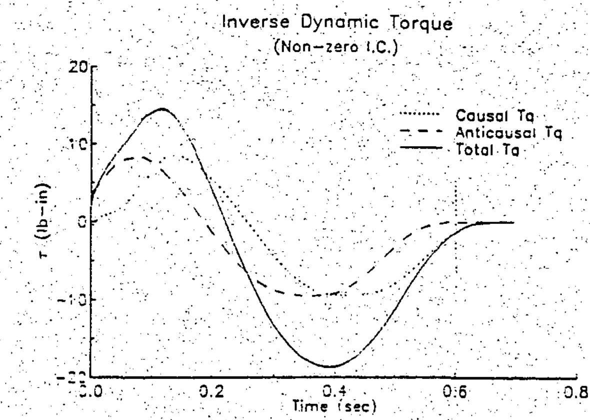

Figure 8. Inverse dynamic torque for non-zero I.C.

7.3 Simulation Studies of Several Initial Tracking Errors

Case 1: Perfect knowledge of I.C. (no initial tracking errors)

In order to get zero initial tracking errors, we have to know and use the exact initial values of the rigid coordinate and causal part flexible coordinate for trajectory planning and torque calculation, and the initial values of the anticausal part flexible coordinates naturally must be the same as the calculated anticausal flexible coordinates at t=0, which were obtained from the backward integration. Then, perfect tracking can be expected as shown in Figure 5.8. The simulation results perfectly match the desired trajectories of the end-point, the joint angle, and strains.

Case 2: Initial errors in flexible coordinates

The assumption of the case 1 is that the initial values of the anticausal flexible coordinates naturally meet with the calculated values of the backward integration. If we relax this unrealistic assumption or allow the measurements of the strain to have errors, we will always have initial tracking errors of flexible coordinates. As the anticausal part torque near t=0 cannot cancel the dynamics of the system, but it is good enough to cancel the dynamics at $t=t_f$, Figure 5.9 shows the asymptotically converging results. Though the strain signals have oscillating tracking errors initially, they converge to the desired trajectories, and produce no overshoot or residual vibrations at the final position.

Case 3: Initial errors in rigid body coordinates

In practical situations, it can be assumed generally that the measurement of the tip acceleration, velocity and position may have noise or errors. In this case, the desired end point trajectory is generated using the mismatched initial conditions. The rigid body coordinate trajectory (equivalent to the end-point trajectory) based on the mismatched I.C., will affect the causal part and the anticausal part torque calculation. Therefore, the feedforward torque cannot achieve the desired tracking characteristics, but the feedback control will compensate the tracking errors asymptotically. As shown in Figure 11, the tip trajectory converges slowly to the desired trajectory. This example was simulated with a wrong estimation of the initial tip velocity 60 in/s for the actual initial tip velocity 100 in/s. Intentionally, large initial errors were used for clear illustration of the tracking convergence.

With this case, we can expect the estimation of the initial value of the rigid body coordinate to be very significant for the inverse dynamic method, because it is used for trajectory planning and it affects the torque calculation directly.

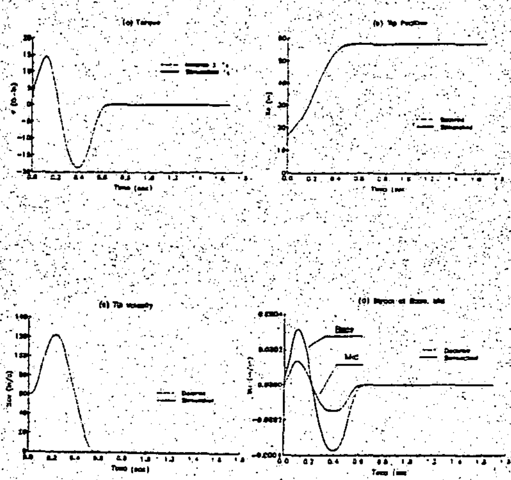

Figure 9. Case 1: Perfect tracking with zero initial tracking errors

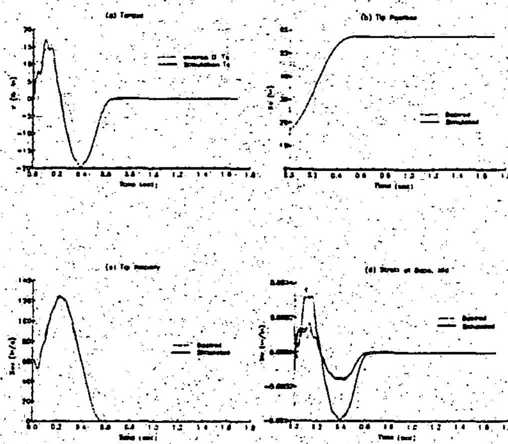

Figure 10. Case 2: Asymptotic tracking with initial errors of flexible coordinates

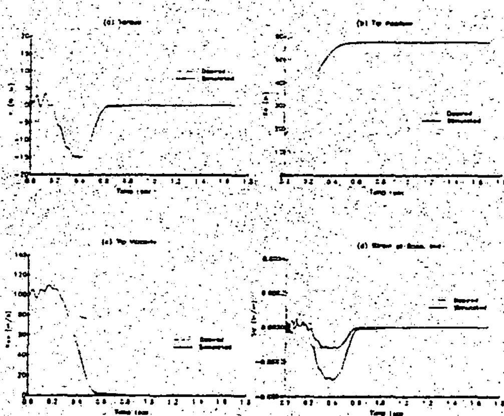

Figure 11. Case 3: Asymptotic tracking with initial errors of rigid body coordinates

7.4 Practical trajectory planning method

The previous simulation analysis is based on the assumption that any calculation of the trajectory planning and the torque and desired trajectory calculation can be performed within one sampling time just after measuring the initial conditions. In order to use this control method for practical application, we had better consider the calculation time to be much longer than one sampling period of the controller, so that a revised trajectory is not immediately available. Moreover, the sudden application of the torque is not desirable because it contains high frequency enough to excite the unmodeled high system natural frequency.

As a practical method, when the actual moving initial conditions are measured at a certain instance, we estimate the states after noncausal time $t=t_0^*$ of Figure 12, and calculate the end point trajectory from the position at $t=t_0^*$ to the final position at $t=t_f$. With this end point trajectory, the inverse dynamic torque and the desired trajectory has been calculated. To avoid the sudden step torque, the noncausal part torque between $t=t_0$ and $t=t_0^*$ is applied as the inverse dynamic torque, assuming the initial flexible coordinate values at $t=t_0$ are all zeros.

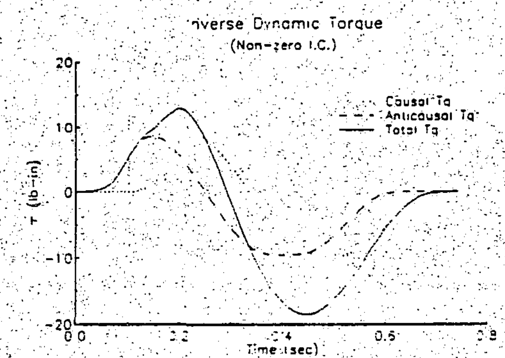

Figure 12. Inverse dynamic torque for non-zero I.C. (Practical approach)

Case 4: Initial errors in general cases (Practical approach)

Since the initial conditions, which were used for trajectory planning and the inverse dynamic calculation, are estimated from the measured initial conditions before $t=t_0^*$, and the flexible coordinate values are assumed as zero, it is natural to have initial tracking errors for all system states and an imperfect inverse dynamic torque profile.

However, Figure 13 clearly shows the asymptotically converging trend of tracking errors, and no overshoot or vibration after $t=t_f^*$.

Case 5: Experimental results of general case

The control scheme of the Case 4 has been implemented to the real experimental flexible manipulator using same conditions. As the end position sensor is not available, the end position behavior can be predicted from the joint angle and the strain measurements. Though the tracking performance in Figure 14 is not as good as the simulation result, the experimental results, which show no overshoot or residual vibrations, are good enough for contact or bracing control. Non-perfect tracking is presumed due to the joint friction because the coulomb friction force deteriorates the inverse dynamic feedforward torque profile especially at the low speed. Despite the initial tracking errors and the friction, the end point stops at the desired position without any overshoot. It indicates the nonminimum-phase system zeros are canceled by the inverse dynamic torque almost perfectly. Thus, this control method for moving initial conditions can be useful to avoid the large impact for contact or bracing.

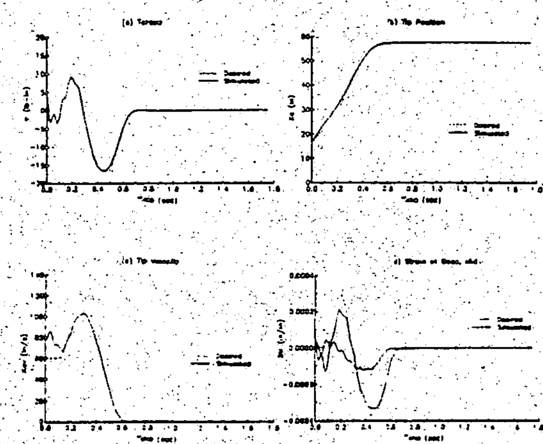

Figure 13. Case 4: Asymptotic tracking for general initial tracking errors

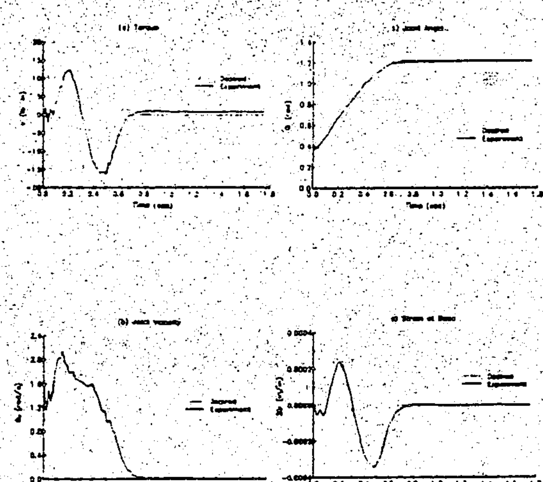

Figure 14. Case 5: Experiment on the tracking control for non-zero I.C.

8. Discussions

This paper presented a time-domain inverse dynamic method that calculates the torque profile and the desired flexible coordinates. It also gives a new interpretation of the inverse system of flexible manipulators. The inverse system has been divided into causal, and anticausal systems and the dynamic system state space can be divided rigid, causal and anticausal system subspaces.

The inverse dynamic torque can be interpreted in two ways. First, it can be considered to generate a feedforward torque to cancel the nonminimum phase system poles and zeros. As a result, the total system transfer function becomes a unit transfer function that makes the output exactly equal to the input. Second, we can interpret the inverse dynamic method as a input shaping technique. It functions like a noncausal notch filter that has zeros at the system resonant frequency. Therefore, the generated input torque profile doesn't contain the system resonant frequency.

Many feedback control schemes require full state feedback, and assigning zero values to the desired flexible mode trajectories has been acceptable for the tracking control of flexible manipulators. Since the time-domain inverse dynamic method provides flexible mode trajectories that match the desired tip trajectory dynamically, it can be used for a trajectory generation method for many advanced feedback control schemes.

If we can measure the I.C. and accommodate the I.C. to the desired trajectories generation, we will obtain perfect tracking with no initial tracking errors eliminating the transient period like the zero I.C. case. However, it is very difficult to accommodate the anticausal part I.C. to the desired trajectory generation because of the backward integration of the anticausal system. This paper's trajectory generation method with the fixed structure acceleration profile will generally have initial tracking errors because of the anticausal part. However, the tracking controller shows the fast tracking convergence in the simulation and experiments. At the final stop, no overshoot nor residual vibration was observed, so that this tracking method will be useful to avoid a large impact for contact or bracing.

Acknowledgement

This research has been partially supported by the National Aeronautics and Space Administration under the Grant No. NAG 1-623.

References

[1] Alberts, T.E., Book, W.J., and Dickerson, S.L., "Experiments in Augmenting Active Control of a Flexible Structure with Passive Damping," AIAA Paper 86-0176, AIAA 24th Aerospace Sciences Meeting, Reno, Nevada, January 6-9, 1986.

[2] Asada, H., and Ma, Z., "Inverse Dynamics of Flexible Robots," *Proceedings of American Control Conference*, 1989, pp.2352-2359.

[3] Bartolini, G., and Ferrara, A., "Extension to Nonminimum Phase Systems of Adaptive Pole Assignment Control by Means of Robustness techniques," *Proceedings of the 29th IEEE Conference on Decision and Control*, December, 1990, pp.994-995.

[4] Bayo, E., "A Finite Element Approach to Control the End-Point Motion of a Single-Link Flexible Robot," *Journal of Robotic Systems*, Vol.4, No.1, 1987, pp.63-75.

[5] Bayo, E., and Paden, B., "On Trajectory Generation for Flexible Robots," *Journal of Robotic Systems*, Vol.4, No.2, 1987, pp.229-235.

[6] Cannon, R.H., and Schmitz, E., "Initial Experiments on the End-point Control of a Flexible One-Link Robot," *The International Journal of Robotic Research*, Vol.3, No.3, Fall, 1984, pp.62-75.

[7] Cetinkunt, S., and Wu, S., "Tip Position Control of a Flexible One Arm Robot with Predictive Adaptive Output Feedback Implemented with Lattice Filter Parameter Identifier," *Proceedings of 1990 IEEE Conference on Robotics and Automation*, pp.1620-1625.

[8] Crawley, E. F., and de Luis, J., "Experimental Verification of Distributed Piezoelectric Actuators for Use in Precision Space Structure," Structures, Structural Dynamics and Materials Conference, San Antonio, Texas, May, 1986.

[9] Hastings, G., and Book, W.J., "Experiments in the Optimal Control of a Flexible Manipulator," *Proceedings of American Control Conference*, Boston, 1985, pp.728-729.

[10] Khorrami, F., and Ozguner, U., "Perturbation Methods in Control of Flexible Link Manipulators," *Proceedings of 1988 IEEE Conference on Robotics and Automation*.

[11] Kotnik, P.T., Yurkovich, S., and Ozguner U., "Acceleration feedback for Control of a Flexible Manipulator Arm," *Journal of Robotic Systems*, 5(3), 1988, pp.181-196.

[12] Kwon, D.-S., and Book, W.J., "An Inverse Dynamic Method Yielding Flexible Manipulator State Trajectories," *Proceedings of American Control Conference*, San Diego, June, 1990, pp.186-193.

[13] Kwon, D.-S., and Book, W.J., "A Framework for Analysis of a Bracing Manipulator with Staged Positioning," *Symposium on Robotics*, Proceedings of ASME Winter Annual Meeting, Chicago, IL, November, 1988, pp.27-36.

[14] Meirovitch, L., Baruh, H., and Oz, H., "A Comparison of Control Techniques for Large Flexible Systems," *AIAA Journal of Guidance*, Vol.6, No.4, July-August, 1983, pp.302-310.

[15] Park, J.H., and Asada, H., "Design and Control of Minimum-Phase Flexible Arms with Torque Transmission Mechanisms," *Proceedings of 1990 IEEE Conference on Robotics and Automation*, Cincinnati, Ohio, May, 1990, pp.1790-1795.

[16] Siciliano, B., and Book, W.J., "A Singular Perturbation Approach to Control of Lightweight Flexible Manipulators," *The International Journal of Robotics Research*, Vol.7, No.4, August, 1988, pp.79-90.

[17] Singer, N.C., and Seering, W.P., "Design and Comparison of Command Shaping Methods for Controlling Residual Vibration," *Proceedings of IEEE Conference on Robotics and Automation*, Scottsdale, AZ, May, 1989.

[18] Singhose, W., Seering, W.P., and Singer, N.C., "Shaping Inputs to Reduce Vibration: A Vector Diagram Approach," *Proceedings of IEEE Conference on Robotics and Automation*, 1990, pp.922-927.

[19] Wang, D., and Vidyasagar, M., "Modelling and Control of Flexible Beam Using the Stable Factorization Approach," *Robotics: Theory and Applications*, Proceedings of ASME Winter Annual Meeting, December, 1986, pp.31-37.

[20] Yuan, B.-S., Book, W.J., and Siciliano, B., "Direct Adaptive Control of a One-Link Flexible Arm with tracking," *Journal of Robotic Systems*, 6(6), 1989, pp.663-680.

[21] Yuh, J., "Application of Discrete-Time Model Reference Adaptive Control to a Flexible Single-Link Robot," *Journal of Robotic Systems*, 4(5), 1987, pp.621-630.

[22] Kwon, D.-S., *An Inverse Dynamic Tracking Control of a Flexible Manipulator for Bracing*, Ph.D. Thesis, Georgia Institute of Technology, June, 1991.